Executive Summary–Team Panchreston

Statement of Problem

Simple organisms must make a quick decision when faced with a potential predator–to flee or not to flee? Using ODE's, we seek to create a model that predicts whether or not an organism should flee from a potential predator given the size of the predator and the rate at which the size is changing. We take a probabilistic approach to this problem: a zero indicates the simple organism does not flee and a one indicates the organism flees.

I. Assumptions and Model

We assume that initially the prey (simple organism) has zero probability of fleeing and its size is given by S_0 in meters^3. Our model is time dependent with time given in seconds. Here we assume that the interaction between the predator and the prey takes place over a ten second interval and the predator is attacking the prey. The size of the predator, relative to the prey, is given by $S(t) = \frac{1}{3}t^2$ (also in m^3) and we assume that the predator is at least 33 m away. We define y to be a function that is proportional to the probability that the prey flees at time, t; thus implying that $\frac{dy}{dt}$ is proportional to the rate at which the prey decides to flee. Note that the proportionality is necessary since our initial model does not return a probability so we had to "normalize" our results using a scaling function, here we use $\frac{1}{2}(1 + \tanh(x - 3.453))$, where 3.453 gives our model numerical stability. Finally, we incorporated a "learning rate", λ, that is equal to the number of times the prey has seen the predator. Putting all of these pieces together, we created the following model:

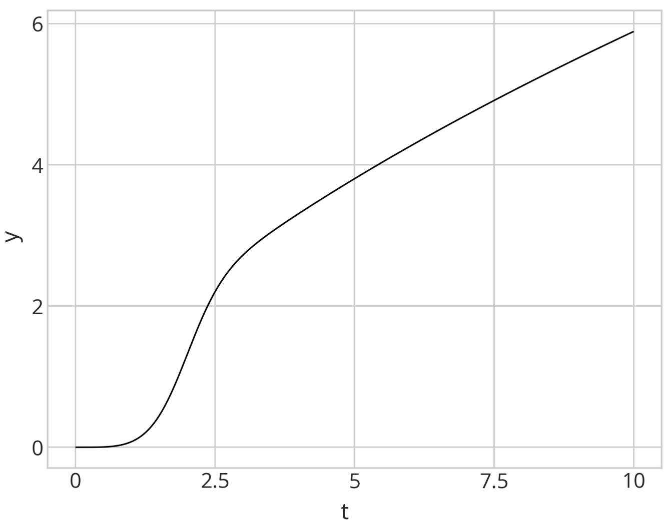
$$\frac{dy}{dt} = \lambda\Big(S'(t)\ln(1 + |\frac{S(t)}{S_0}|) + y\Big) + y(1 - y)). \tag{1}$$

To translate the above ODE, we assumed that the rate at which a prey decides to flee or not is proportionate to the preys current probability of fleeing and the relative size of the predator (notice that the units will cancel in $\frac{S(t)}{S_0}$, thus this ODE is unit–less) times the rate at which the predator is moving at. The ending term of $y(1 - y)$ is a classical logistic model [2] that provides numerical stability.

II. Analysis of the Model

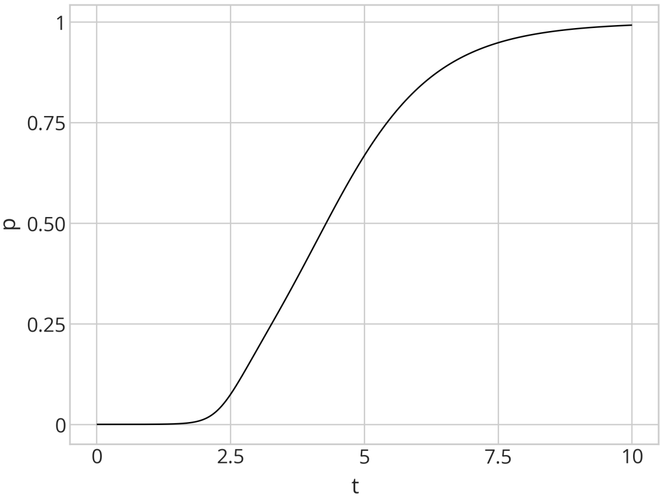
While analytic solutions to equation (1) may be possible to obtain, we chose instead to focus on numerical results. Using 4^{th} Order Runge–Kutta methods [1] in Python, with a range of ten seconds divided equally into 10,001 steps and $\lambda = 1$ (i.e. this is the first time the prey has seen a potential predator) the following plot is produced:

Fig. 1. $Y(t)$ versus time



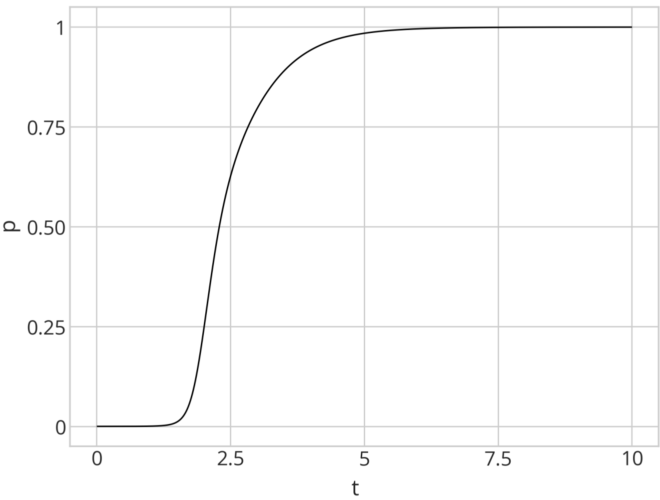
Notice the range of this graph is $[0,6]$ however, we seek a probability. Thus applying the scaling function mentioned above, we receive the following plot:

Fig. 2. Probability versus time



To illustrate how λ affects our model, we change $\lambda = 2$ (i.e. this it the second encounter with the predator) to receive the following plot:

Fig. 3. Probability versus time



Here we can see how increasing λ directly resulted in the prey making a quicker decision.

III. Conclusions

To conclude, equation (1) does an excellent job of modeling a scenario in which a predator is attacking the prey. The parameters $S(t), S_0,$ and λ can all be changed to model new scenarios, however, the results from such changes have not been analyzed yet.

References

[1] Burden, Richard; Faires, J. Douglas. "Numerical Analysis (9^{th} Edition)." (2010). Cengage Learning.
[2] Mooney, Douglas; Swift, Randall. "A Course in Mathematical Modeling." (1999). The Mathematical Association of America.